НАУЧНЫЙ ЖУРНАЛ
НАУКА И МИРОВОЗЗРЕНИЕ

УДК-519.86

THE IMPORTANCE OF MATHEMATICAL MODELLING: A FOUNDATIONAL PILLAR OF MODERN SCIENCE AND ENGINEERING

Ayjeren Atajanova
Department of Applied Mathematics and Informatics, Oguz han Engineering and Technology University of Turkmenistan
Ashgabat, Turkmenistan

Guljeren Tirkeshova
Department of Applied Mathematics and Informatics, Oguz han Engineering and Technology University of Turkmenistan
Ashgabat, Turkmenistan

Abstract

This article conducts an in-depth exploration of the fundamental and increasingly vital role of **mathematical modelling** in modern science, engineering, and society. It defines a mathematical model as a structured, abstract representation of a real-world system using a precise mathematical language. The paper comprehensively outlines the key stages of the modelling process—from **problem formulation** and **simplification** to **model construction**, **analysis**, and **validation**. It distinguishes between different types of models, including deterministic and stochastic, and highlights their significance in providing profound insights, generating robust predictions, and supporting complex decision-making. The discussion demonstrates how mathematical models enable a deeper understanding of phenomena, from the spread of epidemics to the complexities of financial markets, thereby revolutionizing fields ranging from physics and biology to economics and climate science.

Keywords: mathematical modelling, systems analysis, applied mathematics, simulation, prediction, decision-making, scientific research, deterministic models, stochastic models, optimization.

Introduction

In an increasingly complex and interconnected world, the ability to understand, predict, and control real-world systems is paramount. From designing a new airliner to forecasting the path of a hurricane or predicting the spread of a pandemic, we rely on a powerful and versatile tool: **mathematical modelling**. At its core, a mathematical model is a simplified, abstract representation of a real-world process or phenomenon using mathematical language, such as equations, algorithms, and logical relationships. This approach allows us to analyze a system's behavior without the need for expensive, time-consuming, or even impossible physical experimentation.

The true power of modelling lies not just in its predictive capacity, but in its ability to reveal the underlying mechanisms and relationships that govern a system. The purpose of this article is to elucidate the importance of mathematical modelling by examining its core principles, detailed applications across various disciplines, and its transformative impact on problem-solving.

The Process of Mathematical Modelling: An Iterative Cycle

The creation of a valid and useful mathematical model is a structured, iterative process that demands both technical skill and creative insight.

1. Problem Formulation and Simplification

This initial stage is the most critical and arguably the most creative part of the entire mathematical modeling process. Its purpose is to transform a complex, real-world problem into a clear, manageable mathematical task. This is the **art of idealization**, where you intentionally abstract away non-essential details to focus on what truly matters.

Defining the Problem and Key Variables

First, you need to clearly articulate what you want to investigate or predict. For example: "How does a virus spread through a population?" or "What will a stock price be in one month?" Then, you identify the **key variables** that influence the system (e.g., the number of infected individuals, the virus transmission rate, the stock price, interest rates) and the **parameters** (constant values, such as initial population size or dividend amount). This is like creating a list of ingredients before you start cooking.

Idealization and Simplifying Assumptions

No real-world process can be described with 100% mathematical accuracy because it's always influenced by an infinite number of factors. For instance, the trajectory of a thrown baseball isn't just affected by gravity and initial velocity; it's also influenced by air resistance, the ball's spin, humidity, and even the Earth's magnetic field.

Simplification is the process of ignoring these secondary factors. In the baseball example, a physicist might neglect air resistance to create a simple, solvable model based solely on Newton's laws. This assumption makes the model less precise but provides a fundamental understanding of the motion.

The **criteria for good assumptions** are crucial: a good assumption must be realistic enough not to distort the system's essential dynamics, yet simple enough to keep the model solvable. The goal is to find the right balance between **accuracy** and **practicality**. If your simplifications are too radical, your model won't reflect reality. If they are too few, the model will be too complex to solve.

2. Model Construction

Once the problem is simplified, the next step is to translate the conceptual model into a formal mathematical framework. This involves selecting the appropriate mathematical tools to describe the relationships between variables.

Deterministic vs. Stochastic Models: Models can be classified as either **deterministic** or **stochastic**. A deterministic model assumes a predictable outcome for a given set of inputs, with no element of randomness (e.g., a population growth model based on a fixed birth rate). A stochastic model, on the other hand, incorporates randomness and uncertainty to account for unpredictable variables (e.g., a financial model that uses random variables to simulate stock price fluctuations).

Mathematical Tools: The relationships are expressed as equations or a set of logical rules. Common tools include:

Ordinary Differential Equations (ODEs): Used to describe how a system changes over time (e.g., the rate of a chemical reaction).

Partial Differential Equations (PDEs): Used for systems that change over both time and space (e.g., heat diffusion through a material).

Statistical Models: Used to find correlations and relationships within data (e.g., using regression to predict a person's height based on their parents' heights).

3. Model Analysis and Solution

With the model constructed, the next phase is to analyze its behavior and find a solution. This can be done through two primary methods:

Analytical Solutions: This involves finding an exact, closed-form solution to the equations. While highly desirable, this is often possible only for relatively simple models.

Numerical Solutions: For most complex, real-world models, an exact solution is impossible. In these cases, numerical methods are used, which employ computational algorithms to approximate a solution. This stage relies heavily on powerful software and computing power to solve vast systems of equations and generate a solution.

Sensitivity Analysis: This critical step involves systematically varying the model's parameters to see how sensitive the output is to these changes. It helps identify which assumptions or variables have the biggest impact on the results, thereby providing deeper insights into the system itself.

4. Model Validation and Refinement

The final and most crucial stage is to validate the model by comparing its predictions with real-world data.

Validation: If the model's predictions align with observed data, it is considered validated. This often involves using a separate, independent dataset that was not used to build the model. Metrics such as **mean squared error** or **R-squared** are used to quantify the model's accuracy.

Refinement: If the model's predictions do not align with reality, the modeller must revisit the initial assumptions, modify the model, and repeat the analysis. This is an iterative loop that helps the modeller learn more about the system and improve the model's accuracy.

Applications and Importance

Mathematical modelling is an indispensable tool across a vast range of fields due to its ability to provide predictive power and deep insights, often at a lower cost and with greater speed than physical experiments.

In Physics and Engineering: Models are used to design everything from aircraft wings and skyscrapers to electrical circuits. For example, **finite element analysis (FEA)** is used to simulate the stress on a bridge under different loads before it is ever built, ensuring safety and optimizing material usage. This approach dramatically reduces the need for expensive and time-consuming physical prototypes.

In Biology and Medicine: Models help us understand biological systems and processes that are difficult to observe directly. They can simulate the spread of epidemics (like COVID-19), the growth of tumors, or the interaction of drugs with the human body in a process known as **pharmacokinetics**. This accelerates drug discovery and informs public health policy.

In Economics and Finance: Models are used to forecast economic trends, analyze market behavior, and evaluate financial risk. The **Black-Scholes model**, for instance, is a cornerstone of modern financial theory used to price options. In risk management, **Monte Carlo simulations** use random sampling to model the probability of different outcomes, helping institutions manage complex portfolios and hedge against risk.

In Climate and Environmental Science: Complex climate models predict global temperature changes, sea-level rise, and the impact of pollution by incorporating vast amounts of data on atmospheric pressure, ocean currents, and solar radiation. These models are essential for informing policy decisions and resource management, such as using **fisheries models** to set quotas and prevent overfishing.

Conclusion

Mathematical modelling is a powerful and versatile discipline that forms the backbone of modern scientific inquiry and problem-solving. By abstracting the complexities of the real world into a manageable mathematical form, it provides an invaluable framework for understanding, predicting, and making informed decisions. As technology advances and we face increasingly complex global challenges, the importance of mathematical modelling will only continue to grow. It is not merely a tool for calculation; it is a way of thinking, a method for discovering hidden truths, and a means for shaping a better, more predictable future.

Bibliography

1. Bender, E. A. **An Introduction to Mathematical Modeling.** Dover Publications, 2000.
2. Black, F., & Scholes, M. S. **The pricing of options and corporate liabilities.** The Journal of Political Economy, 1973.
3. Brauer, F., & Castillo-Chavez, C. **Mathematical Models in Population Biology and Epidemiology.** Springer, 2012.
4. Edwards, C. H., & Penney, D. E. **Differential Equations and Boundary Value Problems: Computing and Modeling.** Pearson, 2015.
5. Giordano, F. R., Weir, M. D., & Fox, W. P. **A First Course in Mathematical Modeling.** Brooks/Cole, 2013.
6. Sutton, R. S., & Barto, A. G. **Reinforcement Learning: An Introduction.** The MIT Press, 2018.
7. Trefethen, L. N., & Bau, D. **Numerical Linear Algebra.** SIAM, 1997.
8. Sterman, J. D. **Business Dynamics: Systems Thinking and Modeling for a Complex World.** McGraw-Hill/Irwin, 2000.
9. Rachev, S. T. **Handbook of computational and mathematical finance.** Springer Science & Business Media, 2008.
10. Randall, D. A. **An introduction to climate modeling.** In **Physics of Climate.** American Institute of Physics, 2010.