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USING FIBONACCI NUMBERS IN MATHEMATICAL PROBLEMS

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The Fibonacci sequence

Fibonacci published in the year 1202 his now famous rabbit puzzle:

Aman put a male-female pair of newly born rabbits in a field. Rabbits take a month to mature before mating. One month after mating, females give birth to one male-female pair and then mate again. No rabbits die. How many rabbit pairs are there after one year?

To solve, we construct Table 1.1. At the start of each month, the number of juvenile pairs, adult pairs, and total number of pairs are shown. At the start of January, one pair of juvenile rabbits is introduced into the population. At the start of February, this pair of rabbits has matured. At the start of March, this pair has given birth to a new pair of juvenile rabbits. And so on.

month	J	F	M	A	Μ	J	J	A	S	0	Ν	D	J
juvenile	1	0	1	1	2	3	5	8	13	21	34	55	89
adult	0	1	1	2	3	5	8	13	21	34	55	89	144
total	1	1	2	3	5	8	13	21	34	55	89	144	233

Table 1.1: Fibonacci's rabbit population.

We define the Fibonacci numbers F_n to be the total number of rabbit pairs at the start of the nth month. The number of rabbits pairs at the start of the 13th month, $F_{13} = 233$, can be taken as the solution to Fibonacci's puzzle.

Further examination of the Fibonacci numbers listed in Table 1.1, reveals that these numbers satisfy the recursion relation

$$F_{n+1} = F_n + F_{n-1}. \tag{1.1}$$

This recursion relation gives the next Fibonacci number as the sum of the preceeding two numbers. To start the recursion, we need to specify F_1 and F_2 . In Fibonacci's rabbit problem, the initial month starts with only one rabbit pair so that $F_1 = 1$. And this initial rabbit pair is newborn and takes one month to mature before mating so $F_2 = 1$.

The first few Fibonacci numbers, read from the table, are given by

1, 1,2,3,5,8,13,21,34,55,89,144,233,...

and has become one of the most famous sequences in mathematics.

The Fibonacci sequence redux

We can solve another puzzle that also leads to the Fibonacci sequence:

How many ways can one climb a staircase with n steps, taking one or two steps at a time?

Any single climb can be represented by a string of ones and twos which sum to n. We define a_n as the number of different strings that sum to n. In Table 1, we list the possible strings for the first five values of n. It appears that the a_n 's form the beginning of the Fibonacci sequence.

To derive a relationship between a_n and the Fibonacci numbers, consider the set of strings that sum to n. This set may be divided into two nonoverlapping subsets: those strings that start with one and those strings that start with two. For the subset of strings that start with one, the remaining part of the string must sum to n-1; for the subset of strings that start with two, the remaining part of the string must sum to n 2. Therefore, the number of strings that sum to n is equal to the number of strings that sum to n-1 plus the number of strings that sum to n-2. The number of strings that sum to n-1 is given by a_{n-1} and the number of strings that sum to n-2 is given by a_{n-2} , so that

 $a_n = a_{n-1} + a_{n-2}$

And from the table we have $a_1 = 1 = F_2$ and $a_2 = 2 = F_3$, so that $a_n = F_{n+1}$ for all positive integers n

п	strings	a _n
1	1	1
2	11, 2	2
3	111, 12, 21	3
4	1111, 112, 121, 211, 22	5
5	11111, 1112, 1121, 1211, 2111, 122, 212, 221	8